Probability and Random variables

- Signal Processing - extracting information from observed data

- Statistical signal processing – extracting information using statistical tools
- Steps:
  – Signal modelling by random processes – Estimating model parameters
  – Signal estimation by optimal linear filters – Power spectrum estimation
  – Signal detection

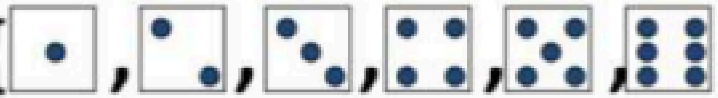
Definition of Probability

Probability is the study of randomness and uncertainty of an experiment. It is a numerical measure of the likelihood that an event will occur, which is expressed as a number between 0 and 1.

Sample Space

The set of all possible outcomes of the experiment, which are assumed equally likely.

E.g. In rolling a die, outcomes are

S ={  }

Event

A sub-set of Sample space of Random Experiment

Probability of an Event

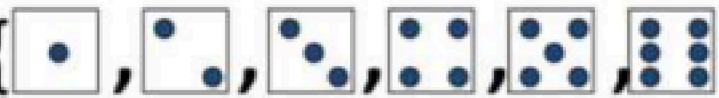
Suppose that the sample space $S = \{o_1, o_2, o_3, \ldots o_N\}$ has a finite number, N, of outcomes. Also each of the outcomes is equally likely.

Then for any event E

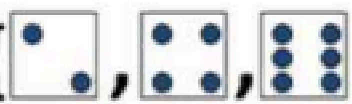
$$P[E] = \frac{n(E)}{n(S)} = \frac{n(E)}{N} = \frac{\text{no. of outcomes in } E}{\text{total no. of outcomes}}$$

- Find the probability of getting an even number when a die is thrown.

When a die is thrown the sample space is

S ={}

The total number of possible outcomes is 6

The favourable number of outcomes is 3, that is {}

$\therefore$ The required probability is= $\frac{3}{6} = \frac{1}{2}$

Some Important Formulas

1. If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

3. If A and A^c are complementary events, then

$$P(A) + P(A^c) = 1$$

4. $P(S) = 1$

5. $P(\Phi) = 0$

- A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace?

Let A be the event of drawing a spade

B be the event of drawing a ace

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13}$$

- In tossing a coin what is the probability of getting Head or tail?

Let A be the event of getting Head

B be the event of getting Tail

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Conditional Probability

The conditional probability of an event B, assuming that the event A has happened, is denoted by P(B/A) and defined as

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \text{provided } \mathrm{P(A)} \neq 0$$

Product theorem of probability

Rewriting the definition of conditional probability, We get

$$P(A \cap B) = P(A)P(B/A)$$

- The probability that a regularly scheduled flight depa
on time is P(D) = 0.83; the probability that it arrives
time is P(A) = 0.82; and the probability that it depa
and arrives on time is $P(D \cap A) = 0.78$. Find t
probability that a plane

  (a) arrives on time, given that it departed on time, an

  (b) departed on time, given that it has arrived on tim